Quotients of the shift map (for frogs)

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The shift map σ on $\omega^* = \beta\omega - \omega$ is the continuous self-map of ω^* induced by the function $n \to n+1$ on ω. Given a compact Hausdorff space X and a continuous function $f : X \to X$, we say that (X, f) is a quotient of (ω^*, σ) whenever there is a continuous surjection $Q : \omega^* \to X$ such that $Q \circ \sigma = \sigma \circ f$.

For spaces of weight at most $\aleph_1$, (X, f) is a quotient of (ω^*, σ) if and only if f is weakly incompressible (which means that no nontrivial open $U \subseteq X$ has $f(\overline{U}) \subseteq U$). Under CH, this gives a complete characterization of the quotients of (ω^*, σ) and implies, for example, that (ω^*, σ^{-1}) is a quotient of (ω^*, σ).

In this talk, I will outline and discuss the proof of this theorem. The main tools used are transfinite recursion, elementary submodels, and a detailed look at the quotients of (ω^*, σ) and $(\beta\omega, \sigma)$.

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