Near-rings of Continuous Functions and Primeness

Geoff Booth[1]

geoff.booth@nmmu.ac.za

If $(G, +)$ is a topological group, define

$$N(G) := \{a : G \to G : a \text{ is continuous}\}$$

and

$$N_0(G) := \{a \in N_0(G) : a(0) = 0\}.$$

Then $N(G)$ and $N_0(G)$ are a near-ring and a zero-symmetric near-ring, respectively. In the case that the topology on G is discrete, we will write $M(G)$ and $M_0(G)$, instead of $N(G)$ and $N_0(G)$, respectively. The structure of $M(G)$ and $M_0(G)$ hes been extensively investigated for almost as long as near-rings themselves have been studied. The implications of the topology on G for the structures of $N(G)$ and $N_0(G)$ have been investigated since the early 1970's.

Several different definitions of primeness exist in the literature of near-rings, all of which generalise the classical concept for associative rings. All of these definitions give rise to prime radicals in the varieties of zero-symmetric and all near-rings.

In this presentation we will investigate the effect of the topology on G on the primeness of $N_0(G)$, and the associated radicals. In particular, we will consider the effects of connectedness, arcwise connectedness and 0-dimensionality. It turns out that the topology has a profound effect, and the results can be very different for the same G with different topologies.

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