Topological Groups, Coset Spaces, and their Remainders

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Suppose G is a topological group and H is a closed subgroup of G. Then G/H is the quotient space of G, that is, members of G/H are left cosets xH, where $x \in G$, and the topology is the quotient topology. The space G/H is homogeneous. "A space" stands for "a Tychonoff space". A space X is *metric-friendly* if there exists a σ-compact subspace Y of X such that $X \setminus U$ is a Lindelöf p-space, for every open neighbourhood U of Y in X, and the following two conditions are satisfied:

1. For every countable subset A of X, the closure of A in X is a Lindel p-space.
2. For every subset A of X such that $|A| \leq 2^\omega$, the closure of A in X is a Lindelöf Σ-space.

Theorem *Every remainder of any paracompact p-space is metric-friendly.*

A coset space $X = G/H$ is *compactly-fibered* if H is compact.

Theorem *For every compactly-fibered coset space $X = G/H$, either each remainder of X is metric-friendly, or each remainder of X is pseudocompact.*

Theorem *Suppose X is a compactly-fibered coset space, and $Y = bX \setminus X$ is a remainder of X. Then the following conditions are equivalent: (1) Y is metacompact; (2) Y is paralindelöf; (3) Y is Dieudonné complete; (4) Y is Lindelöf; (5) Y is metric-friendly.*